Maths Required Before Entering 😷

Solutions to Example Problems

Solution to Example 1:

We need to simplify this fraction into one term. The common denominator in this case is 99.

To get all terms to have a denominator of **99**, we can multiply the **first term by 9/9**, the **second term by 99/99**, and the **third term by 11/11**. Then, we can combine all three terms together.

$$\begin{aligned}
\frac{xy^2}{11} + xy^2 - \frac{xy^2}{9} &= \left(\frac{xy^2}{11}\right)\left(\frac{9}{9}\right) + \left(xy^2\right)\left(\frac{99}{99}\right) - \left(\frac{xy^2}{9}\right)\left(\frac{11}{11}\right) \\
&= \frac{9xy^2}{99} + \frac{99xy^2}{99} - \frac{11xy^2}{99} \\
&= \frac{9xy^2 + 99xy^2 - 11xy^2}{99} \\
&= \frac{97xy^2}{99}
\end{aligned}$$

Solution to Example 2:

The desired variable *x* is stuck in the exponent of an **exponential function.** To rescue *x* 🤠, we need to do several steps:

1. Isolate the exponential function on one side of the equation.
2. Apply the exponential function's **inverse function to both sides of the equation**. We can use the inverse function to extract everything that is "stuck" inside of the exponential function.
3. Rearrange and solve for *x*.

All exponential functions have a corresponding inverse function that will “reverse” them. In the case of e^x, its inverse function is $\ln x$. For us, this means that we need to apply the natural logarithm to our isolated exponential function to “reverse” its effects and rescue the term stuck inside the exponent.

LET’S DO THIS. 😳

Step 1: Isolate the exponential function on one side.

$$e^{2x+1} = \frac{3}{8}$$

Step 2: Apply the inverse function to **both sides of the equation**. When we do this, the exponential function and its inverse **cancel each other out**, and we successfully *rescue* the term that was trapped!

$$\ln\left(e^{2x+1}\right) = \ln\left(\frac{3}{8}\right)$$
$$2x+1 = \ln\left(\frac{3}{8}\right)$$

Step 3: Solve for *x*.

$$2x = \ln\left(\frac{3}{8}\right) - 1$$
$$x = \frac{\ln(3/8) - 1}{2}$$

Solution to Example 3:

The desired variable *x* is stuck in a **logarithmic function**. To rescue *x* 🤠, we need to do several steps:

1. Isolate the logarithmic function on one side of the equation (*already done for us!*).
2. Apply the logarithmic function's **inverse function to both sides of the equation**. We can use the inverse function to extract everything that is "stuck" inside of the logarithmic function.
3. Rearrange and solve for *x*.

All logarithmic functions have a corresponding inverse function that will "reverse" them. In the case of $\ln x$, its inverse function is e^x. For us, this means that we need to apply the natural exponential function to our isolated logarithmic function to "reverse" its effects and rescue the term stuck inside the parentheses.

LET'S DO THIS. 😳

Step 1: Isolate the log term on one side (*already done!*).

$$5 = \ln(2-3x)$$

Step 2: Apply the inverse function to **both sides of the equation**. When we do this, the logarithmic function and its inverse **cancel each other out**, and we successfully *rescue* the term that was trapped!

$$\begin{aligned} e^5 &= e^{\ln(2-3x)} \\ &= 2-3x \end{aligned}$$

Step 3: Solve for *x*.

$$e^5 - 2 = -3x$$
$$x = \frac{e^5 - 2}{-3}$$
$$= \frac{2 - e^5}{3}$$

Solution to Example 4:

First, we can split the graphs into two groups:

1. Lines – Graphs A and D
2. Parabolas – Graphs B and C

We can do the same with the equations:

1. Lines – Equations II and IV
2. Parabolas – Equations I and III

Let's figure out the lines, first!

Graph A has a **negative slope**, and so does Equation IV. Match!

Graph D has a **positive slope**, and so does Equation II. Match!

Now we can figure out the parabolas.

Graph B and C have the same axes, so we can compare the relative "steepness" of the parabolas. Graph B is a skinnier/"steeper" parabola, so we can expect that it should have a larger coefficient in front of the x^2 term.

Also, we can see that Graph C has a vertex (i.e., bottom of the parabola) at a positive *y*-value, while Graph B appears to have a vertex at a negative *y*-value.

Therefore, Graph B matches Equation III, and Graph C matches Equation I.

Summary:

Graph A and Equation IV

Graph B and Equation III

Graph C and Equation I

Graph D and Equation II

Solution to Example 5:

First, we can split the graphs into two groups:

1. Exponential functions with bases that are **greater than 1** – Graphs C and D
2. Exponential functions with bases that are **between 0 and 1** – Graphs A and B

We can do the same with the equations:

1. Exponential functions with bases that are **greater than 1** – Equations I and II
2. Exponential functions with bases that are **between 0 and 1** – Equations III and IV

Let's work with the first group:

Note the following: *the* ***larger*** *the base value, the faster the exponential function grows*.

So, we can expect that the equation $y = 5^x$ will "grow faster" than $y = 3^x$ since 5 > 3.

Therefore, Graph C matches with Equation II and Graph D matches with Equation I.

Let's work with the second group:

Note: *the **smaller** the fraction, the faster the exponential function decays towards zero.*

So, we can expect that the equation $y = \left(\frac{1}{5}\right)^x$ will decay faster towards zero than $y = \left(\frac{1}{3}\right)^x$. However, it's hard to tell between graphs A and B which of the two is "decaying faster towards zero".

Instead, another way to compare the two graphs is to compare their mirror images!

Note: *the mirror image of the graph with a fractional exponent gives the reciprocal exponential function.*

The mirror image $y = \left(\frac{1}{5}\right)^x$ gives the graph of $y = 5^x$. Since Graph D corresponds to $y = 5^x$, then Graph A must correspond to $y = \left(\frac{1}{5}\right)^x$.

Similarly, the mirror image of $y=\left(\frac{1}{3}\right)^x$ gives the graph of $y=3^x$.

Since Graph C corresponds to $y=3^x$, then Graph B must correspond to $y=\left(\frac{1}{3}\right)^x$.

<u>Summary:</u>

Graph A and Equation III

Graph B and Equation IV

Graph C and Equation II

Graph D and Equation I

Solution to Example 6:

a) The function approaches *y* = 2.20 as *x* approaches 1 from the left. Mathematically, $\lim\limits_{x \to 1^-} f(x) = 2.20$.

b) The function approaches *y* = 0.91 as *x* approaches 1 from the right. Mathematically, $\lim\limits_{x \to 1^+} f(x) = 0.91$.

c) No, the limit **does not exist** (DNE) because the left- and right-handed limits are not equal.